Dynamical Gauge-Higgs Unification (SUSY04)[a]

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ABSTRACT

Dynamical gauge-Higgs unification is presented in higher dimensional gauge theory, in which both adjoint and fundamental Higgs fields are a part of gauge fields. Dynamical gauge symmetry breaking is induced through the Hosotani mechanism. Gauge theory, including the $U(3) \times U(3)$ model, is examined on $M^4 \times (T^2/Z_2)$.

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1. Gauge-Higgs unification

Gauge theory in higher dimensions has been studied extensively in which Higgs bosons in four dimensions can be identified with extra-dimensional components of gauge fields. The idea of unifying Higgs scalar fields with gauge fields was first put forward by Manton.[1] In the $SU(3)$ or G_2 gauge theory on $M^4 \times S^2$, the gauge symmetry breaks down to the electroweak $SU(2)_L \times U(1)_Y$ by nonvanishing field strengths on S^2, which at the same time induce the instability due to the nonvanishing energy density.

A better scenario is to start with gauge theory on non-simply connected space. It was shown that dynamics of Wilson line phases, which at the classical level give vanishing energies, can induce gauge symmetry breaking at the quantum level.[2,3] Adjoint Higgs fields in grand unified theories (GUT) are identified with extra-dimensional components of gauge fields. Dynamical symmetry breaking such as $SU(5) \to SU(3) \times SU(2) \times U(1)$ takes place at the quantum level.

Recently the scenario has been elaborated on orbifolds in which boundary conditions play an additional role. Chiral fermions are incorporated and the gauge hierarchy problem in GUT is solved.[4,5,6] Different sets of boundary conditions, however, can be equivalent through the Hosotani mechanism. Quantum treatment of Wilson line phases becomes crucial to determine the physical symmetry of the theory.

There are two types of gauge-Higgs unification. [7]

(i) Gauge-adjoint-Higgs unification

In GUT, Higgs fields in the adjoint representation are responsible for reducing gauge symmetry to the standard model symmetry, $SU(3)\times SU(2)\times U(1)$. In higher dimensional gauge theory extra-dimensional components of gauge fields serve as Higgs fields in the adjoint representation in four dimensions at low energies. This is the gauge-adjoint-Higgs unification introduced in ref. [2].

(ii) Gauge-fundamental-Higgs unification

Electroweak symmetry breaking is induced by Higgs fields in the fundamental representation. They have another important role of giving fermions finite masses. To unify a

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scalar field in the fundamental representation with gauge fields, the gauge group has to be enlarged. In Manton's approach,[1] the gauge group is $SU(3)$ or G_2. In GUT one can start with $SU(6)$ which breaks to $SU(3) \times SU(2) \times U(1)^2$. [8]

2. Gauge theory on orbifolds

If the space is non-simply connected, Wilson line phases become physical degrees of freedom. They are dynamical and affect physics. At the classical level Wilson line phases label degenerate vacua. The degeneracy is lifted by quantum effects. If the effective potential of Wilson line phases is minimized at nontrivial values of Wilson line phases, then the rearrangement of gauge symmetry takes place.

Consider $SU(N)$ guage theory on $M^4 \times (T^2/Z_2)$. Let x^μ and $\vec{y} = (y_1, y_2)$ be coordinates of M^4 and T^2, respectively. $\vec{y}$ and $\vec{y} + \vec{l}_a$ $(a = 1, 2)$ are identified on T^2 where $\vec{l}_1 = (2\pi R_1, 0)$ and $\vec{l}_2 = (0, 2\pi R_2)$. The Z_2-orbifolding is obtained by identifying $-\vec{y}$ with $\vec{y}$. There appear four fixed points on T^2/Z_2 under the parity $\vec{y} \to -\vec{y}$; $\vec{z}_0 = \vec{0}$, $\vec{z}_1 = \frac{1}{2}\vec{l}_1$, $\vec{z}_2 = \frac{1}{2}\vec{l}_2$, and $\vec{z}_3 = \frac{1}{2}(\vec{l}_1 + \vec{l}_2)$.

Gauge fields satisfy the following boundary conditions. [9]

$$
\begin{aligned}
& A_M(x, \vec{y} + \vec{l}_a) = U_a A_M(x, \vec{y})\, U_a^\dagger \quad (a = 1, 2), \\
& \begin{pmatrix} A_\mu \\ A_{y_a} \end{pmatrix} (x, \vec{z}_j - \vec{y}) = P_j \begin{pmatrix} A_\mu \\ -A_{y_a} \end{pmatrix} (x, \vec{z}_j + \vec{y})\, P_j^\dagger \quad (j = 0, 1, 2, 3) \\
& U_a, P_j \in SU(N) \quad , \quad [U_1, U_2] = 0 \quad , \quad P_j^\dagger = P_j = P_j^{-1} \ .
\end{aligned} \tag{1}
$$

In gauge theory U_a and P_j need not be the identity matrix. The only requirement is that physical quantities, particularly the Lagrangian density, should be single-valued on T^2/Z_2. Not all of U_a and P_j are independent;

$$
U_a = P_a P_0 \quad , \quad P_3 = P_1 P_0 P_2 = P_2 P_0 P_1 \quad . \tag{2}
$$

Similarly fermion fields satisfy

$$
\begin{aligned}
& \psi(x, \vec{y} + \vec{l}_a) = \eta_0 \eta_a\, T[U_a] \psi(x, \vec{y}) \quad , \\
& \psi(x, \vec{z}_j - \vec{y}) = \eta_j\, T[P_j]\, (i\Gamma^4\Gamma^5) \psi(x, \vec{z}_j + \vec{y}) \qquad (\eta_j = \pm 1) \ .
\end{aligned} \tag{3}
$$

$T[U_a]\psi = U_a\psi$ or $U_a \psi U_a^\dagger$ for ψ in the fundamental or adjoint representation, respectively.

Different sets of boundary conditions can be gauge equivalent. Under a gauge transformation $A'_M = \Omega A_M \Omega^\dagger - (i/g)\Omega \partial_M \Omega^\dagger$, A'_M obeys a new set of boundary conditions $\{P'_j, U'_a\}$ where

$$
\begin{aligned}
P'_j &= \Omega(x, \vec{z}_j - \vec{y})\, P_j\, \Omega(x, \vec{z}_j + \vec{y})^\dagger \ , \\
U'_a &= \Omega(x, \vec{y} + \vec{l}_a)\, U_a\, \Omega(x, \vec{y})^\dagger \ ,
\end{aligned} \tag{4}
$$

provided $\partial_M P'_j = \partial_M U'_a = 0$. The set $\{P'_j\}$ can be different from the set $\{P_j\}$. When the relations in (4) are satisfied, we write $\{P'_j\} \sim \{P_j\}$. This relation is transitive, and

therefore is an equivalence relation. Sets of boundary conditions form equivalence classes of boundary conditions. [3,6,10]

3. The Hosotani mechanism on orbifolds

Wilson line phases are zero modes (x- and $\vec{y}$-independent modes) of extra-dimensional components of gauge fields $A_{y_a} = \sum \frac{1}{2} A^{\alpha}_{y_a} \lambda^{\alpha}$ where $\{\lambda^{\alpha}, P_j\} = 0$ ($j = 0 \sim 3$) and $[A_{y_1}, A_{y_2}] = 0$. The Hosotani mechanism concerning the dynamics of Wilson line phases are summarized as follows. [2,3,6]

1. Wilson line phases are physical degrees of freedom specifying classical vacua.
2. The effective potential $V_{\rm eff}$ for the Wilson line phases is nontrivial at the quantum level. The global minimum of $V_{\rm eff}$ determines the physical vacuum.
3. If $V_{\rm eff}$ is minimized at nontrivial values, gauge symmetry is spontaneously broken or enhanced.
4. Gauge fields and adjoint Higgs fields (zero modes of A_y) are unified.
5. Higgs fields acquire finite masses at the one loop level. Finiteness of the masses is guaranteed by the gauge invariance.
6. Physics is the same within each equivalence class of boundary conditions.
7. Physical symmetry of the theory is determined by matter content.

In this mechanism Higgs fields are identified with extra-dimensional components of gauge fields. The expectation values of Higgs fields are determined dynamically. It provides dynamical gauge-Higgs unification.

4. The $U(3)_S \times U(3)_W$ model of Antoniadis, Benakli and Quiros's

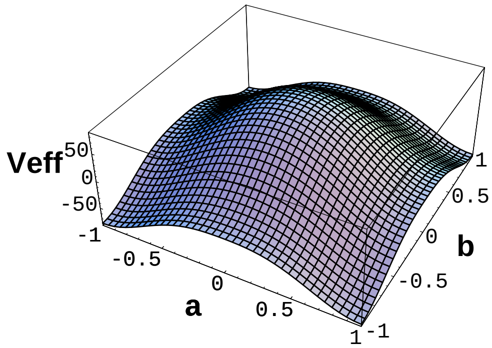
The model [5] is based on a product of two gauge groups $U(3)_S \times U(3)_W$ with gauge couplings g_S and g_W on $M^4 \times (T^2/Z_2)$. $U(3)_S$ is "strong" $U(3)$ which decomposes to color $SU(3)_c$ and $U(1)_3$. $U(3)_W$ is "weak" $U(3)$ which decomposes to weak $SU(3)_W$ and $U(1)_2$. Boundary conditions are given by $P_0 = P_1 = P_2 = I_S \otimes diag\ (-1,-1,1)_W$, which breaks $SU(3)_W$ to $SU(2)_L \times U(1)_1$ at the classical level. By the Green-Schwarz mechanism the symmetry reduces to $SU(3)_c \times SU(2)_L \times U(1)_Y$.

The question to be answered is if the electroweak symmetry breaking occurs by the Hosotani mechanism. There are Wilson line phases in the $SU(3)_W$ group. They are

$$A_{y_1} = \begin{pmatrix} & & \star \\ & & \star \\ \star & \star & \end{pmatrix} = \begin{pmatrix} & \Phi_1 \\ \Phi_1^\dagger & \end{pmatrix} \quad , \quad A_{y_2} = \begin{pmatrix} & \Phi_2 \\ \Phi_2^\dagger & \end{pmatrix} \quad . \tag{5}$$

Φ_1 and Φ_2 are $SU(2)_L$ Higgs doublets. The classical potential has flat directions. Up to $SU(2)$ rotations they are represented by $2g_W R_1 \Phi_1^t = (0, a)$ and $2g_W R_2 \Phi_2^t = (0, b)$. The effective potential $V_{\rm eff}(a, b)$ is nontrivial at the quantum level.

With three generations of quarks and leptons $V_{\rm eff}(a, b)$ is evaluated to be



$$V_{\text{eff}}(a,b) = -40 \cdot I\Big(\frac{a}{2}, \frac{b}{2}\Big) + 4 \cdot I(a,b) \tag{6}$$

where

$$I(a,b) = -\frac{1}{16\pi^2}\left\{ \sum_{n=1}^{\infty} \frac{\cos 2\pi na}{n^6 R_1^6} + \sum_{m=1}^{\infty} \frac{\cos 2\pi mb}{m^6 R_2^6} + \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} \frac{2\cos 2\pi na \cos 2\pi mb}{(n^2 R_1^2 + m^2 R_2^2)^3} \right\} \quad . \tag{7}$$

The global minimum of the effective potential (6) is located at $(a,b) = (1,1)$, which corresponds to the $U(1)_{EM} \times U(1)_Z$ symmetry. Although the $SU(2)_L$ symmetry is partially broken and W bosons acquire masses, Z bosons remain massless. This result is not what we hope to obtain. We would like to have a model in which the global minimum of the effective potential is located at non-integral values of (a,b). Some modification is necessary.

5. Summary

Dynamical gauge-Higgs unification is achieved in higher dimensional gauge theory. Higgs fields are identified with Wilson line phases in gauge theory. Dynamical symmetry breaking is induced by the Hosotani mechanism. Finding a realistic model along this line is awaited.

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